Absorption by fractons in the far infrared

V. N. Novikov and N. N. Ovsyuk

Institute of Mineralogy and Petrography, Siberian Division of the Russian Academy of Sciences, Novosibirsk

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It is shown that the far-infrared absorption spectrum of fractal structures can be used to find a number of fractal parameters of these structures. In particular, the infrared data for polymethylmethacrylate indicate a high degree of superlocalization of vibrations of the high frequency part of the acoustic spectrum.

Fractal systems are attracting greater interest to the physics of disordered materials. They are encountered in the majority of random growth models, including percolation clusters, colloidal aggregates, porous materials, polymers, etc. The dynamic properties of fractal systems are attracting special attention.[1] In particular, the properties of vibrational excitations of fractals (fractons) had been investigated by inelastic neutron and Raman scattering methods.[2] We shall show that the absorption spectra obtained in the far infrared also provide information on fracton parameters.

Our fractal object was the polymer polymethylmethacrylate (PMMA). This material exhibits fractal behavior on a scale of 3-6 nm. In view of the scaling properties of fractals, all the spectra relating to the fractal regime exhibit a power-law frequency dependence. Consequently, in this part of the spectrum the absorption coefficient can be written in the form $\alpha(\omega) \propto \omega^{\mu}$, where the power exponent μ depends on the characteristics of the interaction of light with fractons and on the fracton density of stages $g(\omega)$.

The $\alpha(\omega)$ spectrum was determined experimentally for PMMA using an IFS-113v Fourier spectrometer made by Bruker. The spectrum is shown in Fig. 1 on a double logarithmic scale together with the spectrum of the density of vibrational states $g(\omega)$ determined by the inelastic neutron scattering method.[2] We can see that at frequencies $\omega > 2.5$ meV there is an energy dependence with a slope $\nu = 0.8$ in the case of $g(\omega)$ and with a slope $\mu = 2.2$ in the case of $\alpha(\omega)$.

We can relate the power exponent μ to the fractal characteristics of a material by considering the following model of infrared absorption by fractons. The absorption intensity is described by a matrix element $M(\omega)$ and by the density of vibrational states $g(\omega)$. Since the selection rules break down, all the vibrational modes contribute to the infrared absorption process. Consequently, the absorption coefficient $\alpha(\omega)$ is proportional to the density of vibrational states

$$\alpha(\omega) \propto |M(\omega)|^2 g(\omega), \qquad (1)$$

where the matrix element $M(\omega)$ representing the interaction of light with a fracton localized at a point r is governed by the gradient of the wave function of a fractal and in the continuum approximation can be written in the form

$$M(\omega) = C \int d^D r \ \nabla\phi(r) \ exp\ (i\mathbf{kr}). \qquad (2)$$

Here, exp ($i\mathbf{kr}$) describes a plane light wave and $\phi(r)$ is the wave function of a fracton;

integration is carried out over a fractal structure with a fractal dimensionality D. The constant C is proportional to the elastooptic constant and contains parameters which

depend weakly on the frequency of light. In Eq. (1) an allowance is made for the fact that the oscillatory factor $\omega^{-1/2}$ representing harmonic vibrations is cancelled out by the

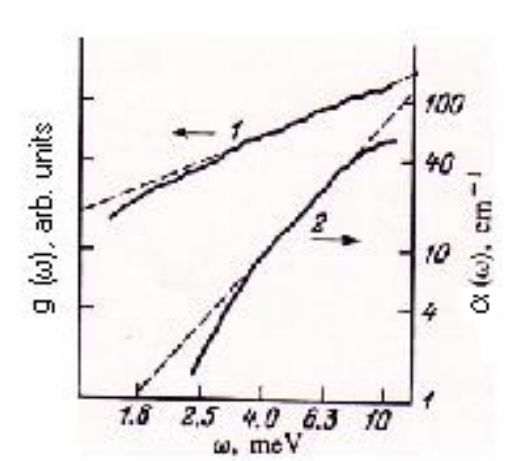

FIG. 1. Density of states (1) and the absorption coefficient (2) of PMMA plotted using log-log coordinates.

same factor $\omega^{1/2}$ representing the electric field of a light wave: $E(\omega) \propto \omega^{1/2} \exp(i\mathbf{kr})$.

We can estimate the matrix element $M(w)$ by adopting the following form of the wave function of a fracton:[3]

$$\phi(r) \propto l_\omega^{-D/2} \exp\left(-(r/l_\omega)^{d_f}\right), \qquad (3)$$

where $l_\omega \propto \omega^{\tilde{d}/D}$ is the fracton localization length; $d_f \geq 1$ *is* a superlocalization factor which arises because an exponential decay of the wave function along a polymer chain corresponds to a stronger decay in terms of the coordinates of the space where the polymer chain is located. Consequently, the matrix element is described by

$$M^2(\omega) \propto \omega^{2\tilde{d}d_f/D - \tilde{d}}. \qquad (4)$$

Bearing in mind that $g(\omega) \propto \omega^{\tilde{d}-1}$, where $\tilde{d}$ is the spectral dimensioinality,[1] we find that the power exponent μ is given by

$$\mu = 2\tilde{d}d_f / D - 1 \qquad (5)$$

which is of the same form as in the case of low-frequency Raman scattering by fractons subject to suitable normalization.[2] The superlocalization parameter can be found from the slope μ. If $\tilde{d} = 1.8$, $D = 2$ (Ref. 2), then we have $d_f \approx 1.8$, which is in reasonable agreement with the value $d_f = 1.5$, found from the Raman scattering of light.[2]

We shall conclude by stating that the far-infrared absorption spectrum of spectral structures can be used to find a number of fractal parameters of these structures. In particular, the infrared data for PMMA indicate a high degree of superlocalization of vibrations of the high-frequency part of the acoustic spectrum.

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